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MR Namitha
Assistant Professor, Department of Agriculture Engineering, Sethu Institute of Technology, Kariapatti, Anna University, Chennai, Tamil Nadu, India

V Vinothkumar
Research Scholar, Department of Farm Machinery and Power, Tamil Nadu Agricultural University, Chennai, Tamil Nadu, India

Correspondence
MR Namitha
Assistant Professor, Department of Agriculture Engineering, Sethu Institute of Technology, Kariapatti, Anna University, Chennai, Tamil Nadu, India

Development of empirical models from rainfall-intensity-duration-frequency curves for consecutive Days maximum rainfall using GEV distribution

MR Namitha and V Vinothkumar

Abstract
A statistical analysis of 27 years rainfall data of Kumulur region in Trichy district of Tamil Nadu was conducted using Gumbel distribution and Generalised Extreme Value (GEV) distribution. The method of L- moments was used for determining the parameters for both distributions. Annual maximum rainfall for 2, 3,4, 5 and 7 consecutive days for the available 27 years were analysed and the return levels for four assumed return periods *viz.* 2, 5, 10 and 25-years were calculated using both the probability distribution functions. The goodness-of-fit of the distributions were analysed by conducting a chi-square test for the observed and expected return levels. The consecutive days maximum rainfall data found to fit best with generalised extreme value distribution. The Empirical Reduction Formula proposed by the Indian Meteorological Department was used for the calculation of short duration for 1-hr, 2-hr, 3-hr, 5-hr, and 8-hr. From the derived short duration rainfall depths, the intensity of the rainfall was calculated. The rainfall Intensity-Duration-Frequency (R-IDF) curves were plotted for the region and the corresponding empirical models were derived.

Keywords: Generalised extreme value distribution, Gumbel distribution, chi-square test, l-moments, r-IDF empirical models

Introduction

Analysis of consecutive Days maximum rainfall of different return periods is a basic tool for safe economic planning and design of small dams, bridges, culverts, irrigation and drainage works. It is possible to predict design rainfall fairly accurately for certain return periods using various probability distributions, even though the rainfall is erratic (Upadhaya and Singh, 1998) [10]. Analysis of consecutive day's maximum rainfall is more relevant for drainage design of agricultural lands (Bhattacharya and Sarkar, 1982; Upadhaya and Singh, 1998). [2, 10] Planning and management of cropping pattern require the analysis of weekly rainfall data.

Different probability distributions *viz.* Gumbel distribution, Log Pearson distribution, log normal distribution, Generalised Extreme Value (GEV) distribution etc. are used for analysing the extreme rainfall events, prediction of return levels and generation of rainfall-intensity-duration-frequency relations.

Materials and Methods

Agriculture Engineering College and Research Institute, Kumulur campus which is located in Lalgudy taluk in Trichy district of Tamil Nadu is chosen as the study area, where the latitude and longitude is 10.55'29.34" N and 78.49'35.61" E. The average annual rainfall of the area was found to be 85.8 cm. The daily rainfall data from 1991 to 2017 was collected from the meteorological observatory of the study area.

The daily data for each year is converted to 2 to 7 consecutive days rainfall by summing up the rainfall of corresponding previous days. The maximum amount of one week and 2 to 5 consecutive days rainfall for each year was taken for the analysis using two probability distribution *viz* Gumbel distribution and Generalised Extreme Value (GEV) distribution. The annual maxima rainfall for the consecutive days were tabulated in table 1. management skills. Substantial increase in productivity Substantial increase in productivity d 0.5 per cent mineral matter. The mineral matter reported to be present in fair amount of calcium, phosphorus, iron, potassium, sodium and iodine.

Table 1: Annual maximum rainfall for 2-7 consecutive days

Sl. No.	Year	Annual Maximum Rainfall for consecutive days				
		2 days	3 days	4 days	5 days	7 days
1	1991	181.5	190	197.4	197.4	197.4
2	1992	137.4	172.7	196.7	225.7	235.6
3	1993	133.8	157.4	175.4	176.4	180.4
4	1994	73.4	88.2	88.2	102.8	113.8
5	1995	130	130	130	130	130
6	1996	122.6	137.1	137.1	151	201.8
7	1997	72.9	106.3	107.3	118.1	134.1
8	1998	136.3	197.2	217	233.3	263.9
9	1999	217.4	219.8	224.4	224.4	227.1
10	2000	177.0	184.4	206	213.4	213.4
11	2001	123	123	123	150.5	189.5
12	2002	208.9	260.1	260.1	260.1	260.1
13	2003	114.2	130.6	131.6	131.6	131.6
14	2004	127.5	163.2	182.2	182.2	206
15	2005	204.9	216.2	254	263	277.8
16	2006	84.2	91.9	91.9	91.9	104.2
17	2007	192	209.2	223	223	246
18	2008	287.5	390.7	419.7	427.2	430.4
19	2009	191.2	191.2	242.2	257.4	257.4
20	2010	139.6	180	186.2	193.6	212.6
21	2011	114	132.8	153.8	157.2	161.4
22	2012	110.6	122.6	130.6	141	168.2
23	2013	84.5	95.2	99.7	99.7	112.8
24	2014	67	70.2	86.6	97.1	98.4
25	2015	75.7	79.7	92.5	97.9	97.9
26	2016	72.9	79.7	88.2	97.1	98.4
27	2017	67	70.2	86.6	91.9	97.9

Table 2: Statistical parameters of annual consecutive Days maximum rainfall

Sl. No.	Parameters	2 day	3 day	4 day	5 day	7 days
1.	Minimum (mm)	67	70.2	86.6	91.9	97.9
2.	Maximum (mm)	287.5	390.7	419.7	427.2	430.4
3.	Mean (mm)	140.28	161.59	174.26	181.84	194.07
4.	Standard deviation, σ (mm)	11.02	13.78	15.07	15.10	15.03
5.	Coefficient of skewness	0.80	1.5	1.38	1.36	1.13
6.	Kurtosis	0.49	3.90	3.29	3.23	2.63

The statistical parameters of consecutive Days maximum rainfall are shown in Table 2.

Gumbel distribution and Generalized Extreme Value distribution were used for the analysis of extreme rainfall events and the calculation of return levels. The calculated 2-7 days maximum rainfall data were fitted to the corresponding probability distribution functions.

Fitting the Distributions for the Extreme Rainfall Analysis

a. Generalized Extreme Value Distribution (GEV)

The cumulative distribution function of GEV is defined in (Hosking, 1997) [5] as:

$$F(x) = \exp\{-\left(1 - \frac{k(x-\xi)}{\alpha}\right)^{\frac{1}{k}}\} \quad \ldots\ (1)$$

where, ξ is the location parameter, α is the scale parameter, and k is the shape parameter.

b. Gumbel Distribution (EV1)

The cumulative distribution function for Gumbel distribution as defined in (Hosking, 1997) [5] is:

$$F(x) = exp\left[-exp\left(-\frac{x-\xi}{\alpha}\right)\right] \quad \ldots\ (2)$$

Where, ξ is the location parameter, α is the scale parameter.

Parameter estimation for the distributions

The method of L-moments was widely used for the determination of parameters of various probability distribution functions, and are also nearly unbiased (Rowinski, 2001) [9]. The annual maximum data must be arranged in ascending order, and the L-moments were calculated by applying the equations proposed by Cunnane (1989) [4].

The estimated parameters of GEV and Gumbel using the method of L-Moments are tabulated in table 3 and table 4.

Table 3: Parameters for GEV distribution

Sl. No	Parameters for GEV	Annual Maximum Rainfall for consecutive days				
		2 days	3 days	4 days	5 days	7 days
1	Scale, α	45.23	51.57	57.09	57.82	62.89
2	Location, ξ	114.70	130.54	105.68	147.30	161.68
3	Shape, k	0.0144	-0.0299	-0.028	-0.019	0.0667

Table 4: Parameters for Gumbel distribution

Sl. No	Parameters for Gumbel	Annual Maximum Rainfall for consecutive days				
		2 days	3 days	4 days	5 days	7 days
1	Scale, α	44.64	53.05	58.65	58.91	59.34
2	Location, ξ	114.52	130.97	106.43	148.46	157.77

The return period for an event can be calculated by the following formula:

$$T = \frac{N+1}{m} \quad \ldots.. (3)$$

Where, N is the total number of years of record and R is the rank of observed rainfall values arranged in descending order. The amount of rainfall equalled or exceeded at the given return period is known as return level. In present study, the return levels of rainfall are calculated for the assumed return periods of 2, 5, 10 and 25 years.

Calculation of Return Levels

With the help of theoretical probability distributions, the rainfall of various magnitudes with different return periods were forecasted. The probability distributions used in this study are Gumbel and Generalised Extreme Value distribution.

Gumbel distribution: The equation for fitting the Gumbel distribution to observed series of flood flows at different return periods T is (Sarma, 1999) [8]:

$$\boldsymbol{X_T = \bar{X} + K\sigma} \quad \ldots.. (4)$$

where, X_T denotes the magnitude of the T- year flood event, K is the frequency factor $\bar{X}$ and σ are the mean and the standard deviation of the maximum instantaneous flows respectively.

The frequency factor is expressed as

$$K = -\frac{\sqrt{6}}{\pi}(0.5772 + \ln\left(ln\frac{T}{T-1}\right)) \quad \ldots.. (5)$$

Generalised Extreme Value Distribution: The cumulative distribution function of the GEV distribution is:

$$X_T = \xi + \frac{\alpha}{k}[1 - (-\ln\left(1 - \frac{1}{T}\right))^k] \quad \ldots.. (6)$$

Where, T is the return period, X_T is the return level at T years.

The observed and the expected return levels for consecutive Days maximum rainfall using both the probability distributions are tabulated in table 5.

Table 5: Observed and Expected return levels for consecutive Days maximum rainfall

S. No.	Return Period, T (Years)		2	5	10	25
1	2 Days Maximum Return Level	Observed	130.00	192.00	208.90	287.50
		Gumbel	138.47	148.21	154.66	162.81
		GEV	131.32	183.28	218.15	262.75
2	3 Days Maximum Return Level	Observed	157.40	209.20	219.80	390.70
		Gumbel	159.32	171.51	179.58	189.77
		GEV	149.55	209.65	250.58	303.63
3	4 Days Maximum Return Level	Observed	175.40	224.40	254.00	419.70
		Gumbel	171.78	185.11	193.93	205.08
		GEV	126.71	193.13	238.29	296.71
4	5 days Maximum Return Level	Observed	176.40	233.30	260.10	427.20
		Gumbel	179.37	192.66	201.46	212.57
		GEV	168.57	235.27	280.24	337.97
5	7 Days Maximum Return Level	Observed	197.40	257.40	277.80	430.40
		Gumbel	191.60	204.89	213.69	224.80
		GEV	185.01	260.89	314.38	385.90

Goodness of fit

The observed and expected rainfall calculated using both probability distribution functions and the calculated values of rainfall were compared to find the goodness of fit using Chi-square test.

$$\chi^2 = \sum_{i=1}^{n}\frac{(O_i - E_i)^2}{E_i} \quad \ldots.. (7)$$

where, O_i is the observed rainfall and E_i is the expected return level using probability distribution functions.

Estimation of Short Duration Rainfall

The short duration rainfall for 1-hr, 2-hr, 3-hr, 5-hr and 8-hr was calculated using empirical reduction formula (eq. 8) proposed by Indian Meteorological Department (IMD).

$$P_t = P_{24}\sqrt[3]{\frac{t}{24}} \quad \ldots.. (8)$$

where, P_t is the required rainfall depth in mm at t-hr duration, P_{24} is the daily rainfall in mm and t is the duration of rainfall for which the rainfall depth is required in hour.

The rainfall intensity for a particular short duration rainfall is calculated by:

$$I_t = \frac{P_t}{t} \quad \ldots.. (9)$$

Where, I_t is the intensity of rainfall in mm h^{-1} for return period T, P_t is the required rainfall depth in mm at t-hr duration and t is the duration in hours.

Development of R-IDF Empirical Relations

The following steps were followed to find the R-IDF empirical equations:

1. The original equation for R-IDF relation (eq. 10) is converted into logarithmic function (Chow (1988); Koutsoyiannis *et al.* (1998), and AlHassoun (2011) [3, 7, 1]) as follows:

$$I = C\frac{T_r{}^m}{T_d{}^e} \qquad \text{…. (10)}$$

Where T_r is the duration of rainfall, T_d is the return period and I is the intensity of rainfall.
By applying logarithmic conversion to eq. (10):

$$\log I = \log K + e \log T_d \qquad \text{…. (11)}$$

Where,

$$K = CT_r{}^m \qquad \text{…. (12)}$$

And e is the slope of the straight line.

2. Plot a graph with log I on y-axis and log T_d on x-axis for all the return periods for all the proposed short durations.
3. The value of e (slope of the straight line) is found for each return period and the average value of e was calculated as:

$$e = \frac{\sum e}{n} \qquad \text{…. (13)}$$

Where, n is the number of return periods under consideration.

4. Then, the log K values (y-intercept) are found out from the graph and the eq. 12 was converted into logarithmic function as:

$$\log K = \log C + m \log T_r \qquad \text{…. (14)}$$

5. Plot the values of log K on y-axis and log T_r on x-axis to find the values of parameters C and m. m is the slope of the straight line and C is the antilog for y-intercept. The final R-IDF relations for each short duration are calculated by applying these parameter values in eq. 10.

Results and Discussions

The return levels of extreme rainfall for 2, 3, 4, 5 and 7-days maxima for 2, 5, 10 and 20 years return period was calculated using the cumulative distribution functions of Gumbel and Generalised Extreme Value (GEV) distributions. The analysis of the observed and estimated return levels using chi-square test revealed that generalised extreme value distribution attains a good fit with the data of the study area. Therefore, it can be concluded that, the rainfall data of the study area fits best with generalised extreme value distribution.

The short duration rainfall depths for 1-hr, 2-hr, 3-hr, 5-hr, and 8-hr were calculated using the empirical reduction formula (eq. 8) proposed by Indian meteorological department. The intensities of corresponding short duration rainfall were calculated using eq. 9. The derived intensities were plotted against duration in a logarithmic scale. The R-IDF curves generated for the study area for 2-7 consecutive Days maximum rainfall were depicted in fig.1 to fig. 5.

The R-IDF empirical models (eq. 10.) for 2-7 consecutive Days maximum rainfall were calculated by the method proposed in eq. 11 to eq. 13. The calculated empirical methods were tabulated in table 6.

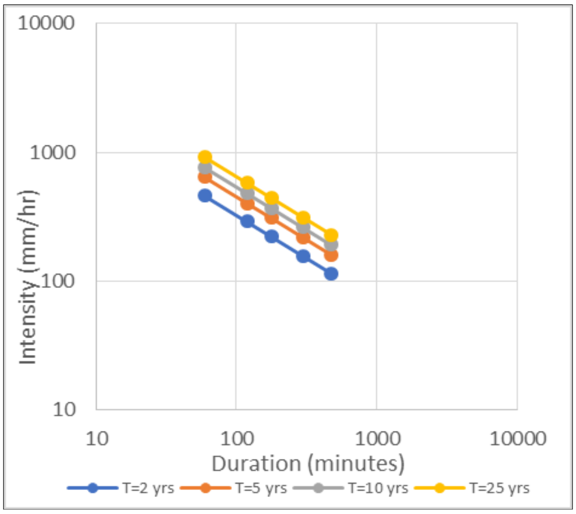

Fig 1: R-IDF curve for 2-day maximum rainfall

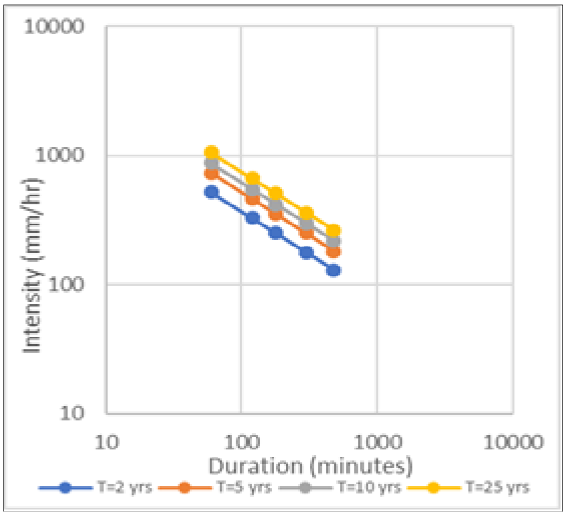

Fig 2: R-IDF curve for 2-day maximum rainfall

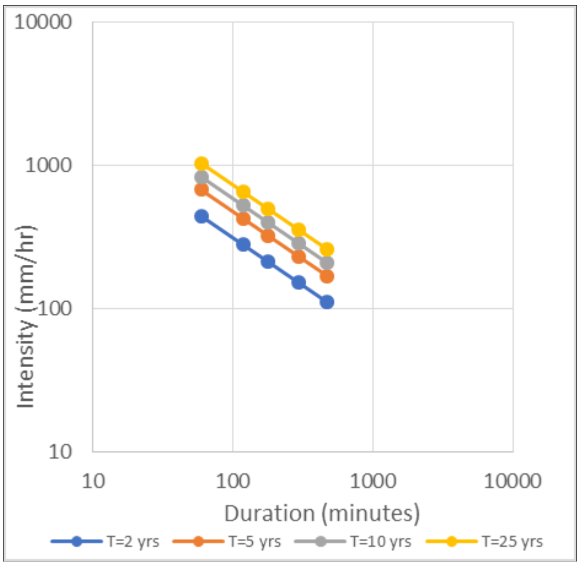

Fig 3: R-IDF curve for 4-day maximum rainfall

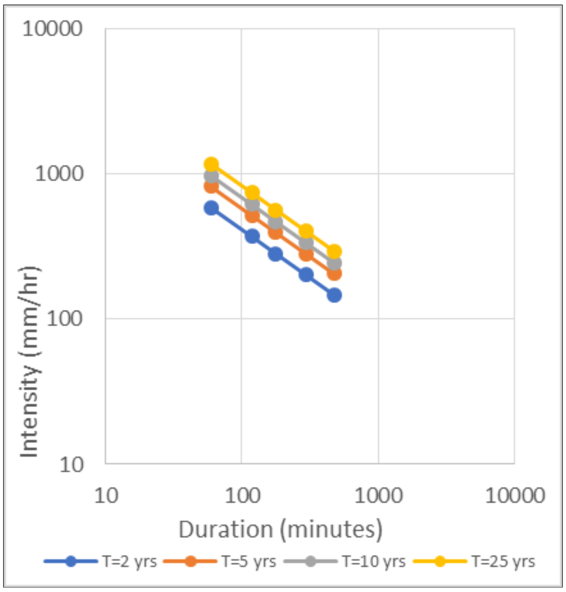

Fig 4: R-IDF curve for 5-day maximum rainfall

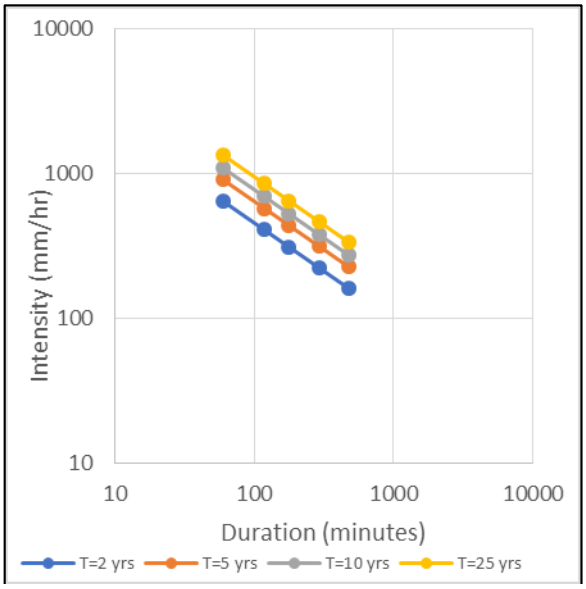

Fig 5: R-IDF curve for 7-day maximum rainfall

Table 6: R-IDF Empirical Relations

S. No.	Consecutive days	R-IDF Empirical Relation
1	2-day maximum rainfall	$I = 6003 \frac{T_r^{0.272}}{T_d^{0.667}}$
2	3-day maximum rainfall	$I = 6806 \frac{T_r^{0.280}}{T_d^{0.667}}$
3	4-day maximum rainfall	$I = 5606 \frac{T_r^{0.338}}{T_d^{0.667}}$
4	5-day maximum rainfall	$I = 7672 \frac{T_r^{0.276}}{T_d^{0.667}}$
5	7-day maximum rainfall	$I = 8321 \frac{T_r^{0.291}}{T_d^{0.667}}$

The analysis of table 6 reveals that the parameters C, and m differs for each empirical model. The value of e remains a constant of 0.667 for all models. Using the above given empirical models the intensity for any duration and return level can be calculated for the study area. The values of e and m shows a good correspondence with the study conducted by Ibrahim (2012) [6] in Saudi Arabia.

Conclusion

The result of the analysis shows that the rainfall of the study area fits best with generalised extreme value distribution. The R-IDF curves and empirical models were generated for the study area which enables an easy analysis of intensity and return levels for different durations in corresponding return periods.

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